Unit-4
Network Functions

Syllabus

Network Functions: Pre- Requisites: Concept of basic circuital law, parallel, series circuits. Concept of complex frequency, Network functions of one port and two port networks, Concept of poles and zeros, Properties of driving point and transfer functions. Two Port Networks Characterization of LTI two port networks; Z, Y,ABCD, g and h parameters, Reciprocity and symmetry, Inter-relationships between the parameters, Inter- connections of two port networks, Ladder and Lattice networks: T & Π representation, terminated two Port networks.

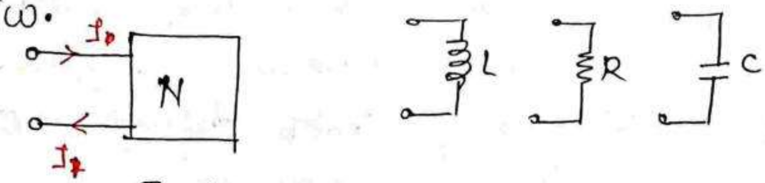
Course Outcome

Demonstrate the concept of complex frequency and analyse the structure and function of one and two port network. Also evaluate and analysis two-port network parameters.

Faculty: Shweta Singh
+91-8392878633; shwetasingh10057@gmail.com

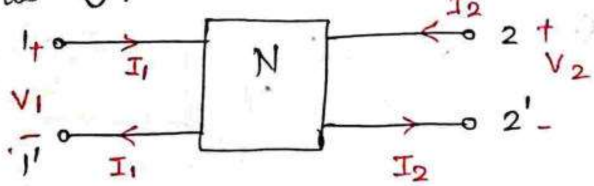
TWO-PORT N/W

- A pair of terminals through which current may enter & leave a n/w is known as a Port.
- Two terminal devices such as R, L, C result in a 1-port n/w.



$I = I_1$

- Current entering 1-terminal of the port leaves from the other so that the net current entering the port is 0.



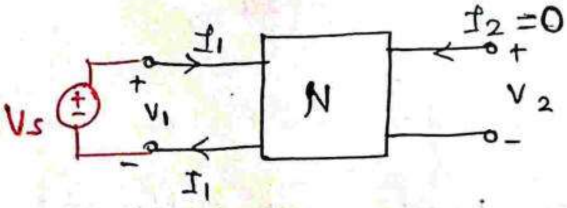
- A 2-port n/w have 2 seperate ports, we apply the i/p at one port & get a o/p from the other. for ex- transformer.
- Current entering both the ports is a standard notation in 2-port n/w.
- In 2-port n/w, we have 4 variables V_1, I_1 for the i/p port & V_2, I_2 for o/p Port. & 2 out of these 4 variables will be dependent & rest of 2 will be independent.
- The n/w inside the ports is considered as a black box. The n/w inside the ports is linear, bidirectional & passive & it consist of linear, bidirectional & passive elements.
- The black box may also consist of energy storage elements like Inductor & Capacitor but their initial conditions must be 0.

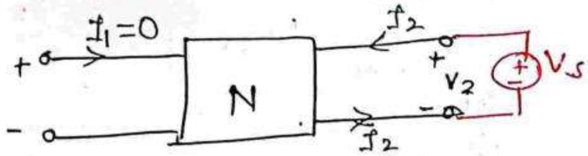
- The n/w inside the ports may also consist of dependent source, never an independent source in it.

Concept of Symmetry in 2-Port N/w.

A 2-port n/w is said to be symmetrical if the ratio of excitation to response remain the same at both the ports independently w.r.t defined ckt conditions. such as o/p ckt or short ckt.

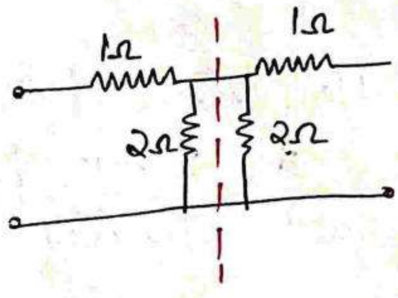
Note for small independent n/w's, symmetry can also be identified as the mirror image property.





$$\left.\frac{V_s}{I_1}\right|_{I_2=0} \qquad \left.\frac{V_s}{I_2}\right|_{I_1=0}$$

for symmetry, $\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}$

- 

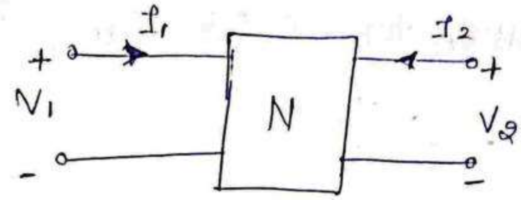
Z-Parameters (or) Open Ckt Parameters (or) Impedance Parameters

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = f\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$V_1, V_2 \rightarrow$ Dependent

$I_1, I_2 \rightarrow$ Independent.

$$\boxed{\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned}}$$



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[V] = [Z][I]$$

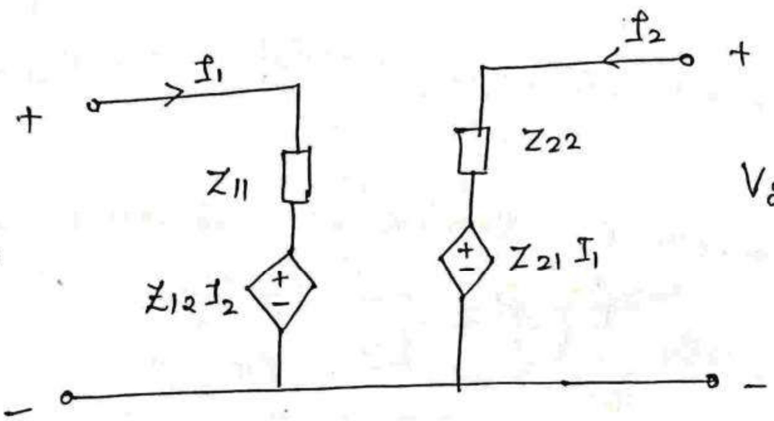
$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$Z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0}$ open ckt driving point i/p Impedance.

$Z_{21} = \left.\dfrac{V_2}{I_1}\right|_{I_2=0}$ open ckt forward Transfer impedance

$Z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0}$ open ckt reverse transfer impedance

$Z_{22} = \left.\dfrac{V_2}{I_2}\right|_{I_1=0}$ open ckt driving Point o/p Impedance



Y-Parameters / Admittance Parameters / Short-ckt Parameters

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = f\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$I_1\ I_2 \longrightarrow$ Dependent

$V_1, V_2 \longrightarrow$ Independent.

$$\boxed{\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 \\ I_2 &= Y_{21}V_1 + Y_{22}V_2 \end{aligned}}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$Y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2=0} \neq \dfrac{1}{Z_{11}}$ Short circuit driving point i/p admittance

$Y_{21} = \left.\dfrac{I_2}{V_1}\right|_{V_2=0}$ Short ckt forward Transfer admittance

$Y_{12} = \left.\dfrac{I_1}{V_2}\right|_{V_1=0}$ Short ckt reverse transfer admittance

$Y_{22} = \left.\dfrac{I_2}{V_2}\right|_{V_2=0}$ Short ckt driving point o/p admittance.

$$[I] = [Y][V]$$

$$[V] = [Z][I]$$

$$[I] = [Z]^{-1}[V]$$

$$\left\{ [Z]^{-1} = \frac{\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}}{Z_{11}Z_{22} - Z_{12}Z_{21}} \right\}$$

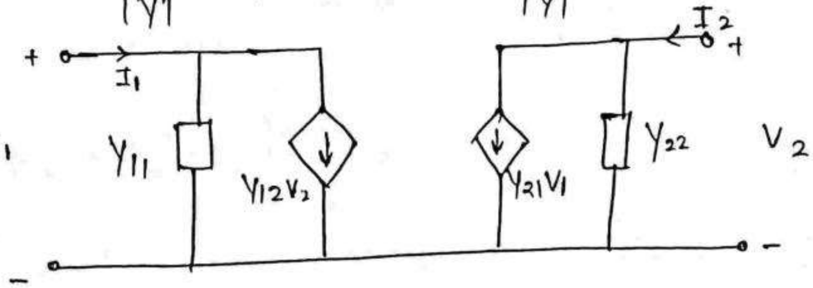
$$\boxed{[Y] = [Z]^{-1}}$$

$$Y_{11} = \frac{Z_{22}}{|Z|} \qquad Y_{12} = \frac{-Z_{12}}{|Z|}$$

$$Y_{21} = \frac{-Z_{21}}{|Z|} \qquad Y_{22} = \frac{Z_{11}}{|Z|}$$

- $$\boxed{[Z] = [Y]^{-1}}$$

$$Z_{11} = \frac{Y_{22}}{|Y|}, \quad Z_{12} = \frac{-Y_{12}}{|Y|}, \quad Z_{21} = \frac{-Y_{21}}{|Y|}, \quad Z_{22} = \frac{Y_{11}}{|Y|}$$



[Electrical equivalent ckt]

H-Parameters

$$\begin{pmatrix} V_1 \\ I_2 \end{pmatrix} = f \begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$$

V_1 I_2 $\longrightarrow$ Dependent

I_1 V_2 $\rightarrow$ Independent.

☆
$$\boxed{\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

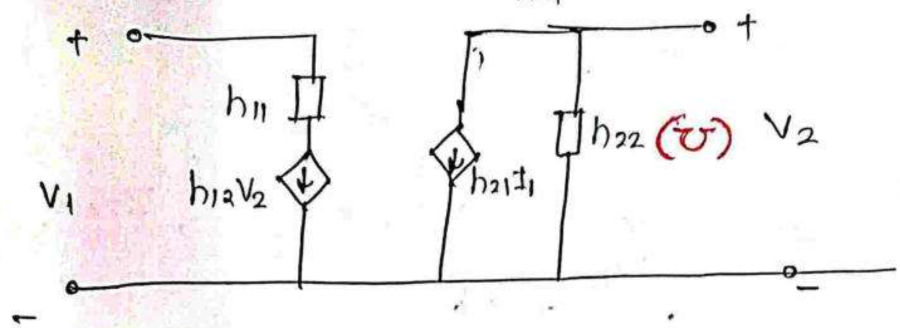
$$[H] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ Short ckt driving point i/p Impedance

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$ Short ckt forward current Gain

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$ open ckt reverse voltage Gain

$$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0} = \frac{1}{\frac{V_2}{I_2}\Big|_{I_1=0}} = \frac{1}{Z_{22}}$$ open ckt driving point o/p admittance



G- Parameters

$$\begin{pmatrix} I_1 \\ V_2 \end{pmatrix} = f\begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$$

$I_1, V_2 \longrightarrow$ Dependent.

$V_1, I_2 \longrightarrow$ Independent.

*
$$\boxed{\begin{aligned} I_1 &= g_{11}V_1 + g_{12}I_2 \\ V_2 &= g_{21}V_1 + g_{22}I_2 \end{aligned}}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$[g] = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

$$[g] = [H]^{-1}$$

$$g_{11} = \frac{h_{22}}{|H|} \qquad g_{12} = \frac{-h_{12}}{|H|}$$

$$g_{21} = \frac{-h_{21}}{|H|} \qquad g_{22} = \frac{h_{11}}{|H|}$$

- $[H] = G^{-1}$

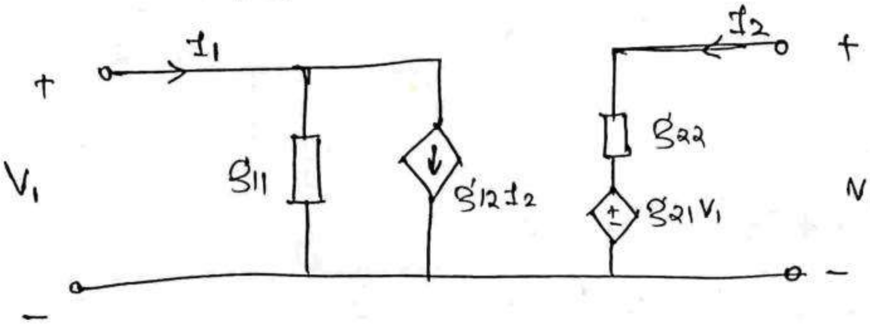
$$h_{11} = \frac{g_{22}}{|G|} \qquad h_{12} = \frac{-g_{12}}{|G|} \qquad h_{21} = \frac{-g_{21}}{|G|} \qquad h_{22} = \frac{g_{11}}{|G|}$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} = \frac{1}{\left.\frac{V_1}{I_1}\right|_{I_2=0}} = \frac{1}{Z_{11}}$$ open ckt driving pt. i/P admittance

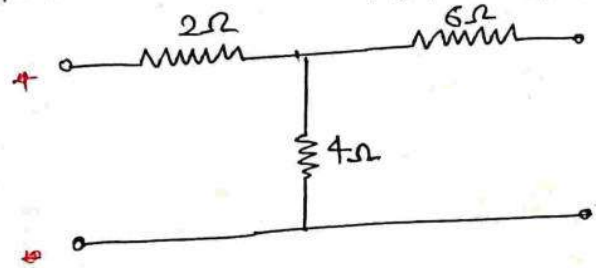
$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$ open ckt forward voltage Gain

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$ Short ckt reverse current Gain

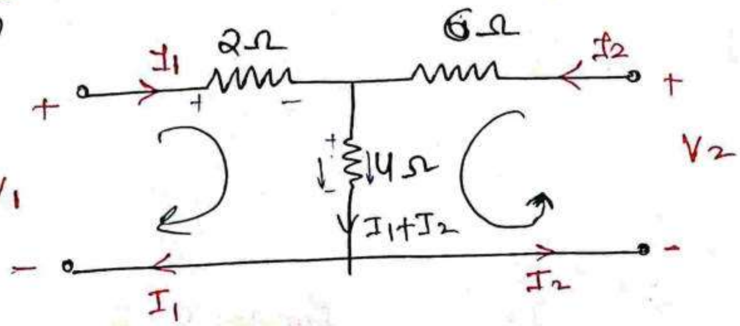
$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = \frac{1}{Y_{22}}$$ short ckt driving point o/p Impedance.



Q. for the below given 2 port n/w Z, Y, h, G Parameters



Soln



Z

$V_1 = 2I_1 + 4(I_1 + I_2)$

$V_1 = 6I_1 + 4I_2$ —— ①

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = 6I_2 + 4(I_1 + I_2)$

$V_2 = 4I_1 + 10I_2$ —— ②

$V_2 = Z_{21}I_1 + Z_{22}I_2$

$$[Z] = \begin{bmatrix} 6 & 4 \\ 4 & 10 \end{bmatrix}$$

② Y: $[Y] = [Z]^{-1}$

$$[Y] = \frac{1}{44}\begin{bmatrix} 10 & -4 \\ -4 & 6 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} \frac{5}{22} & \frac{-1}{11} \\ \frac{-1}{11} & \frac{3}{22} \end{bmatrix}$$

③ H: $\left.\begin{matrix} V_1 \\ I_2 \end{matrix}\right\} \rightarrow I_1 V_2$

② $\rightarrow$ $10 I_2 = -4 I_1 + V_2$

$I_2 = -0.4 I_1 + 0.1 V_2$ ← ③

$I_2 = h_{21} I_1 + h_{22} V_2$

③ → ① $V_1 = 6 I_1 + 4[-0.4 I_1 + 0.1 V_2]$

$V_1 = 4.4 I_1 + 0.4 V_2$ — ④

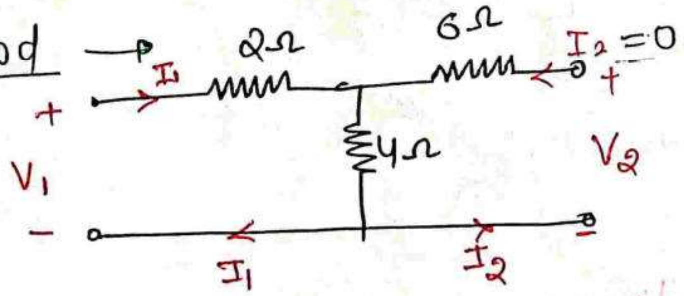
$V_1 = h_{11} I_1 + h_{12} V_2$

$$[H] = \begin{bmatrix} 0.4 & 0.4 \\ -0.4 & 0.1 \end{bmatrix}$$

④ G:

$[G] = [H]^{-1}$

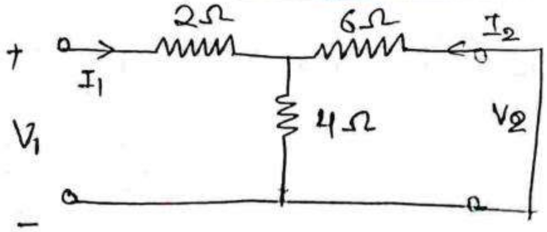
II Method



Put $I_2 = 0$

$V_1 = 6 I_1 \Rightarrow \frac{V_1}{I_1} = 6$

$V_2 = 4 I_1 \Rightarrow \frac{V_2}{I_1} = 4$



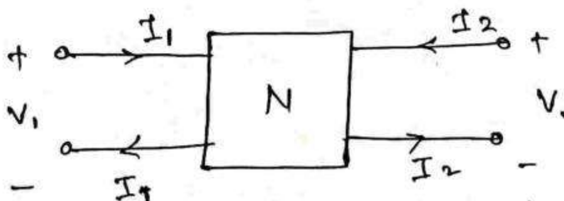
T-Parameters / Transmission Parameters / ABCD Parameters

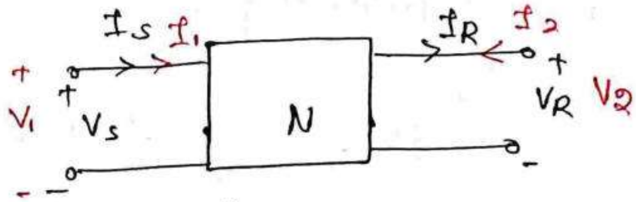
$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = f \begin{pmatrix} V_2 \\ I_2 \end{pmatrix}$$

$V_1, I_1 \longrightarrow$ Dependent

$V_2, I_2 \longrightarrow$ Independent.

$$\boxed{\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}}$$





$V_S = AV_R + BI_R$

$I_S = CV_R + DI_R$

$V_1 = AV_2 + B(-I_2)$

$I_1 = CV_2 + D(-I_2)$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$ open ckt ~~forward~~ Reverse Voltage Gain.

$C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0}$ open ckt ~~driving point i/p~~ Reverse transfer admittance

$B = \left.\dfrac{V_1}{-I_2}\right|_{V_2=0}$ Short ckt ~~reverse driving point~~ reverse Transfer admittance

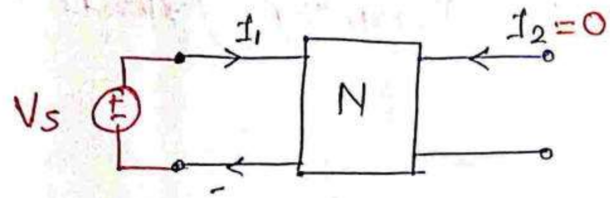
$D = \left.\dfrac{I_1}{-I_2}\right|_{V_2=0}$ Short ckt reverse current Gain.

Condition for Symmetry in 2-Port N/ω :

$V_1 = Z_{11} I_1 + Z_{12} I_2$ — ①

$V_2 = Z_{21} I_1 + Z_{22} I_2$ — ②

1)

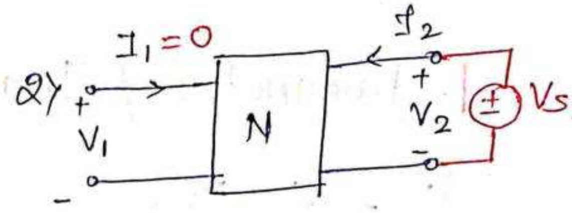


$V_1 = V_s , \; I_2 = 0$

① → $V_s = Z_{11} I_1 + Z_{12} (0)$

$\frac{V_s}{I_1} = Z_{11}$ —— (α)

2)



$V_2 = V_s , \; I_1 = 0$

② → $V_s = Z_{21} (0) + Z_{22} I_2$

$\frac{V_s}{I_2} = Z_{22}$ —— (β)

for symmetry,

$$\left.\frac{V_s}{I_1}\right|_{I_2=0} = \left.\frac{V_s}{I_2}\right|_{I_1=0}$$

from eq^n α & β.

* $\boxed{Z_{11} = Z_{22}}$ Symmetry.

$$\frac{Y_{22}}{|Y|} = \frac{Y_{11}}{|Y|}$$

$\boxed{Y_{11} = Y_{22}}$ *

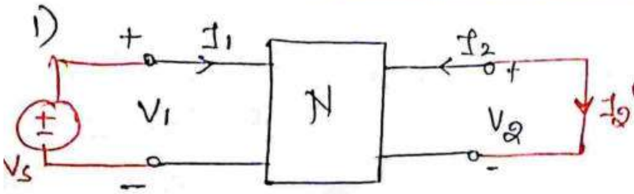
Condition for Reciprocity in 2-Port N/ω :

A N/ω is said to be reciprocal if the ratio of response to excitation remains the same even when the position of response & excitation are interchanged.

$V_1 = Z_{11} I_1 + Z_{12} I_2$ — ①

$V_2 = Z_{21} I_1 + Z_{22} I_2$ — ②

1)



$V_1 = V_s,\ V_2 = 0,\ I_2 = -I_2'$

$$(1) \rightarrow V_s = Z_{11} I_1 + Z_{12}(-I_2') \quad \text{— (3)}$$

$$(2) \rightarrow 0 = Z_{21} I_1 - Z_{22} I_2'$$

$$I_1 = \frac{Z_{22}}{Z_{21}} I_2' \quad \text{— (4)}$$

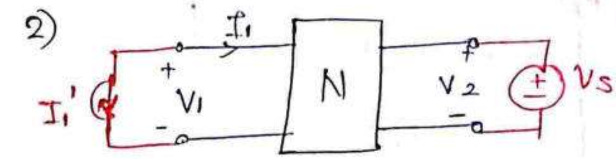
$(4) \rightarrow (3)$

$$V_s = Z_{11}\left(\frac{Z_{22}}{Z_{21}} I_2'\right) - Z_{12} I_2'$$

$$V_s = I_2'\left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}\right]$$

$$\frac{I_2'}{V_s} = \frac{Z_{21}}{|Z|} \quad \text{— } (\alpha)$$

2)



$V_2 = V_s,\ V_1 = 0,\ I_1 = I_1'$

$$(2) \rightarrow V_s = Z_{21} I_1' + Z_{22} I_2 \quad \text{— (5)}$$

$$(1) \rightarrow 0 = -Z_{11} I_1' + Z_{12} I_2$$

$$I_2 = \frac{Z_{11}}{Z_{12}} I_1' \quad \text{— (6)}$$

$(6) \rightarrow (5)$

$$V_s = Z_{21} I_1' + Z_{22}\frac{Z_{11}}{Z_{12}} I_1'$$

$$V_s = I_1'\left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{12}}\right]$$

$$\frac{I_1'}{V_s} = \frac{Z_{12}}{|Z|} \quad \text{— } (\beta)$$

for reciprocity

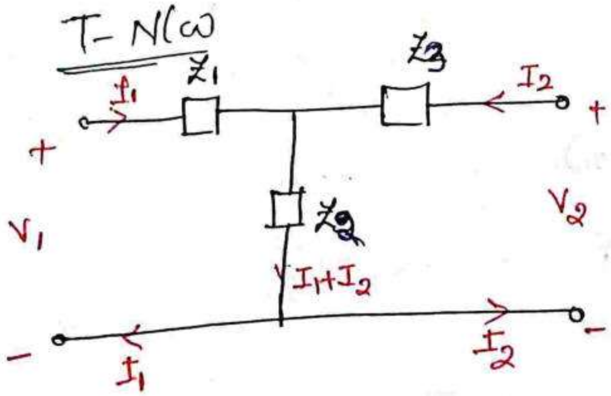
$$\frac{I_1'}{V_s} = \frac{I_2'}{V_s}$$

from eq^n α & β

$$\frac{Z_{12}}{|Z|} = \frac{Z_{21}}{|Z|} \qquad \frac{-Y_{12}}{|Y|} = \frac{-Y_{21}}{|Y|}$$

$$\boxed{Z_{12} = Z_{21}} \text{ Reciprocity} \qquad \boxed{Y_{12} = Y_{21}}$$

T- N/w



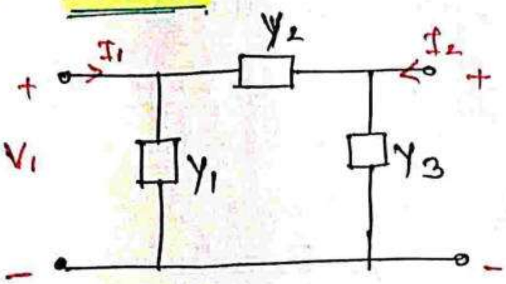
$$V_1 = (Z_1 + Z_2) I_1 + Z_2 I_2$$

$$V_2 = Z_2 I_1 + (Z_2 + Z_3) I_2$$

$$Z_{11} = Z_1 + Z_2$$
$$Z_{12} = Z_{21} = Z_2$$
$$Z_{22} = Z_2 + Z_3$$

$$Z_1 = Z_{11} - Z_{12}$$
$$Z_2 = Z_{12}$$
$$Z_3 = Z_{22} - Z_{12}$$

π- N/ω



- $I_1 = V_1 Y_1 + (V_1 - V_2) Y_2$

$$I_1 = (Y_1 + Y_2) V_1 - Y_2 V_2 \quad \text{— (1)}$$

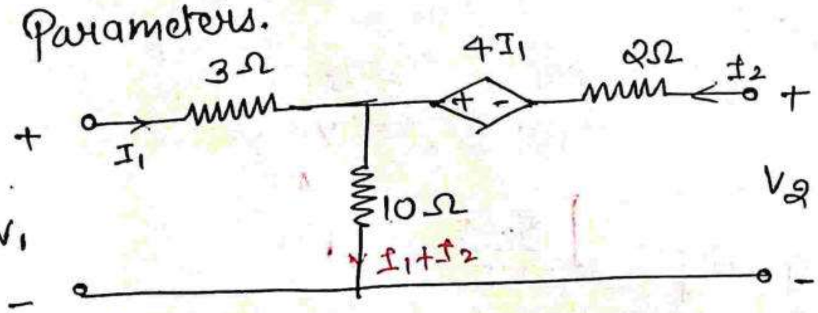
- $I_2 = V_2 Y_2 + (V_2 - V_1) Y_2$

$$I_2 = -Y_2 V_1 + (Y_2 + Y_3) V_2 \quad \text{— (2)}$$

$$Y_{11} = Y_1 + Y_2$$
$$Y_{12} = Y_{21} = -Y_2$$
$$Y_{22} = Y_2 + Y_3$$

$$Y_1 = Y_{11} + Y_{12}$$
$$Y_2 = -Y_{12}$$
$$Y_3 = Y_{22} + Y_{12}$$

Q. In the below given 2 port n/ω, find Z, Y, h, g & T Parameters.



① Mark voltage
② Mark current
③ KVL

Sol^n 1) Z

- $V_1 = 13 I_1 + 10 I_2$ — (1)
- $V_2 = 2I_2 - 4I_1 + 10(I_1 + I_2)$

$$V_2 = 6I_1 + 12 I_2 \quad \text{— (2)}$$

$$[Z] = \begin{bmatrix} 13 & 10 \\ 6 & 12 \end{bmatrix}$$

2) $\underline{\underline{Y}}$ $[Y] = [Z]^{-1}$

$$[Z] = \begin{bmatrix} 13 & 10 \\ 6 & 12 \end{bmatrix} \Rightarrow |Z| = 13\times12 - 60 = 96$$

$$Y_{11} = \frac{+12}{96} \qquad Y_{12} = \frac{-10}{96} \qquad Y_{21} = \frac{-6}{96} \qquad Y_{22} = \frac{+13}{96}$$

$$Y = \begin{bmatrix} \frac{12}{96} & \frac{-10}{96} \\ \frac{-6}{96} & \frac{13}{96} \end{bmatrix}$$

3) $\underline{\underline{H}}$

$$12 I_2 = -6 I_1 + V_2$$

$$I_2 = \frac{-1}{2} I_1 + \frac{1}{12} V_2 \quad \text{—— } (3)$$

$$V_1 = 13 I_1 + 10\left[\frac{-1}{2} I_1 + \frac{1}{12} V_2\right]$$

$$V_1 = 8 I_1 + \frac{5}{6} V_2 \quad \text{—— } (4)$$

$$[H] = \begin{bmatrix} 8 & 5/6 \\ -\frac{1}{2} & \frac{1}{12} \end{bmatrix}$$

4) $\underline{\underline{G}}$, $[G] = [h]^{-1}$

$$|h| = \cancel{8}^{4} \times \frac{1}{\cancel{12}_{3}} + \frac{1}{2}\times\frac{5}{6} = 1.3 + 0.4 = 1.71$$

$$h_{11} = \frac{-8}{1.71} \qquad h_{12} = \frac{5}{6\times1.71} \qquad h_{21} = \frac{-1}{2\times1.71}, \quad h_{22} = \frac{-1}{12\times1}$$

5) $\underline{\underline{T}}$.

$$(2) \rightarrow 6 I_1 = V_2 - 12 I_2$$

$$I_1 = \frac{1}{6} V_2 - 2 I_2 \quad \text{—— } (5)$$

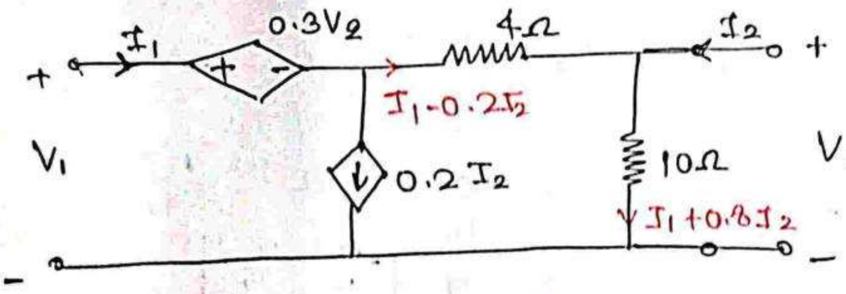
$$(5) \rightarrow (1)$$

$$V_1 = 13\left[\frac{1}{6} V_2 - 2 I_2\right] + 10 I_2$$

$$V_1 = \frac{13}{6} V_2 - 16 I_2 \quad \text{—— } (6)$$

$$[T] = \begin{bmatrix} \frac{13}{6} & 16 \\ \frac{1}{6} & 2 \end{bmatrix}$$

Q. find Z-Parameters.



- $V_2 = 10[I_1 + 0.8 I_2]$

$$V_2 = 10I_1 + 8I_2 \quad —(1)$$

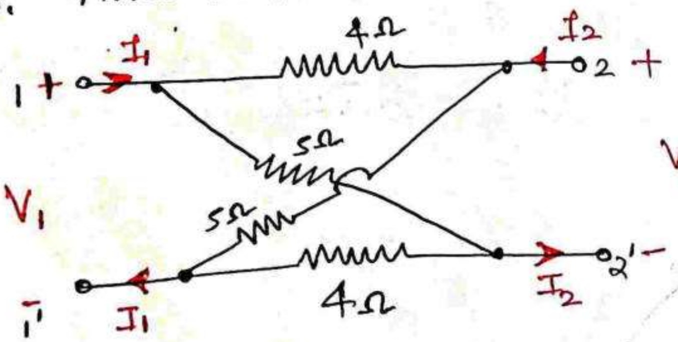
- $V_1 = 0.3V_2 + 4(I_1 - 0.2 I_2) + 10(I_1 + 0.8 I_2)$

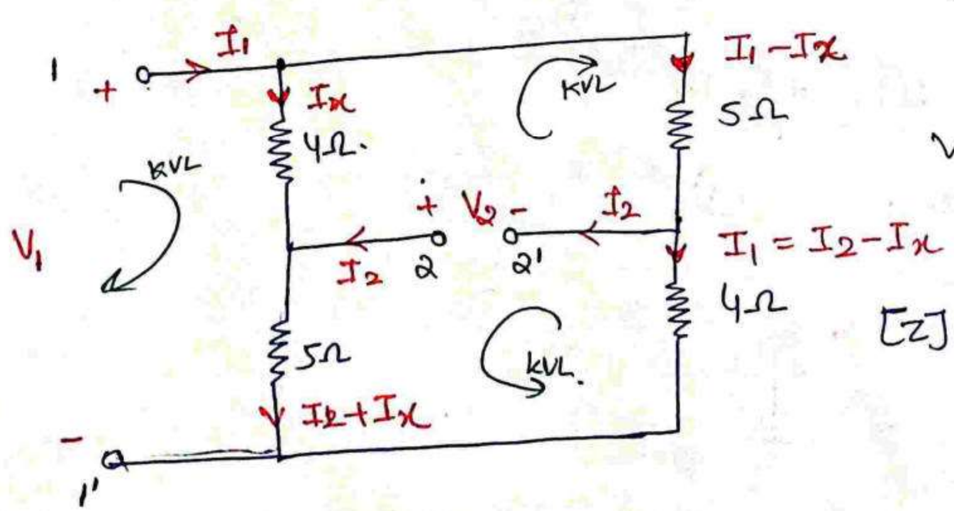
$(1) \rightarrow V_1 = 0.3(10I_1 + 8I_2) + 14I_1 + 7.2I_2$

$$V_1 = 17I_1 + 9.6 I_2 \quad —(2)$$

$$[Z] = \begin{bmatrix} 17 & 9.6 \\ 10 & 8 \end{bmatrix}$$

Q. find Z-Parameters.





- $V_1 = 4I_x + 5(I_2 + I_x)$

$$V_1 = 9I_x + 5I_2 \quad —(1)$$

- $V_2 = -4I_x + 5(I_1 - I_x)$

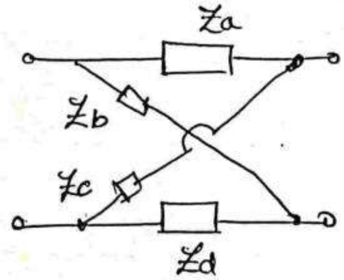
$$V_2 = -9I_x + 5I_1 \quad —(2)$$

$$V_2 = 5(I_2 + I_x) - 4[I_1 - I_2 - I_x]$$

$$V_2 = -4I_1 + 9I_2 + 9I_x \quad —(3)$$

$$[Z] = \begin{bmatrix} 9/2 & \frac{1}{2} \\ 1/2 & \frac{9}{2} \end{bmatrix}$$

#



$\left.\begin{array}{l} Z_a = Z_d \\ Z_b = Z_c \end{array}\right\}$ symmetry.

$$\boxed{\begin{array}{l} Z_{11} = Z_{22} = \dfrac{Z_a + Z_b}{2} \\[2ex] Z_{12} = Z_{21} = \dfrac{Z_b - Z_a}{2} \end{array}}$$

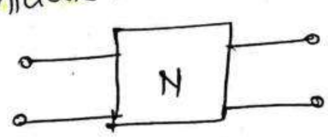
$\dfrac{\text{cross} - \text{straight arm}}{2}$

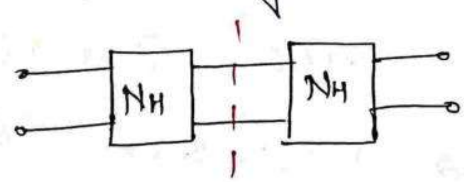
$\left.\begin{array}{l} Y_a = Y_b \\ Y_b = Y_c \end{array}\right\}$ symmetry

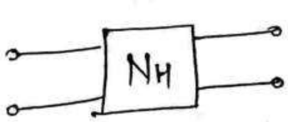
$$\boxed{\begin{array}{l} Y_{11} = Y_{22} = \dfrac{Y_a + \cancel{Y_b}}{2} \\[2ex] Y_{12} = Y_{21} = \dfrac{Y_a - Y_b}{2} \end{array}}$$

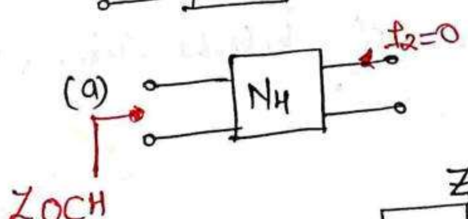
BARTLETT BISECTION THEOREM :

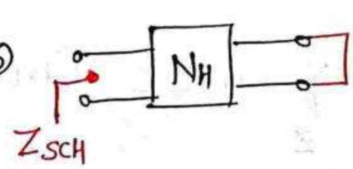
- It is used for the designing of the lattice n/w.
- It is applicable only for the symmetrical n/w.
- In Bartlett bisection theorem, we seperate the given n/w into 2 equal parts & then bisect from the middle.

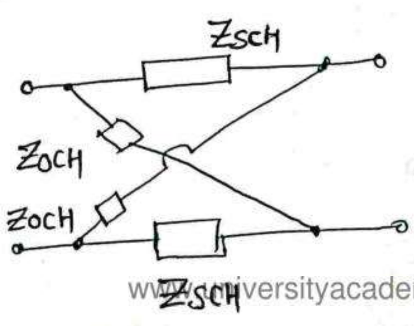
1) 

2) 

3) 

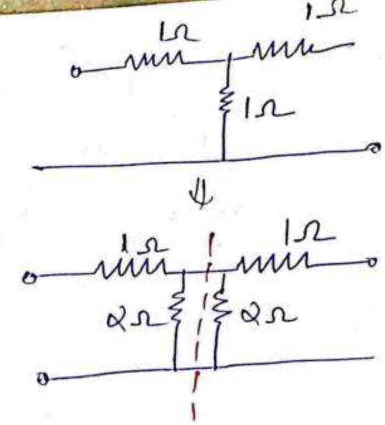
(a) 

(b) 

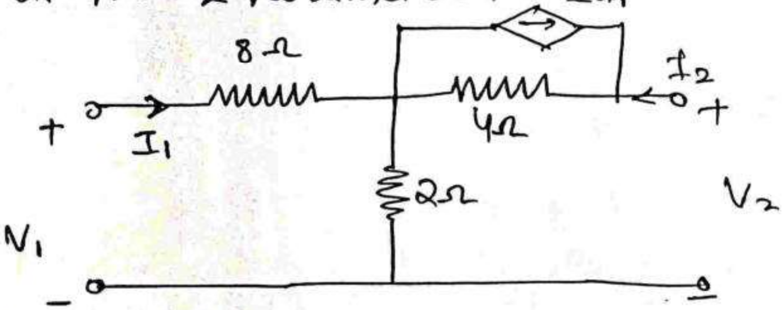


$$\left\{ \begin{array}{l} Z_{11} = Z_{22} = \dfrac{Z_{OCH} + Z_{SCH}}{2} \\ \\ Z_{12} = Z_{21} = \dfrac{Z_{OCH} - Z_{SCH}}{2} \end{array} \right\}$$

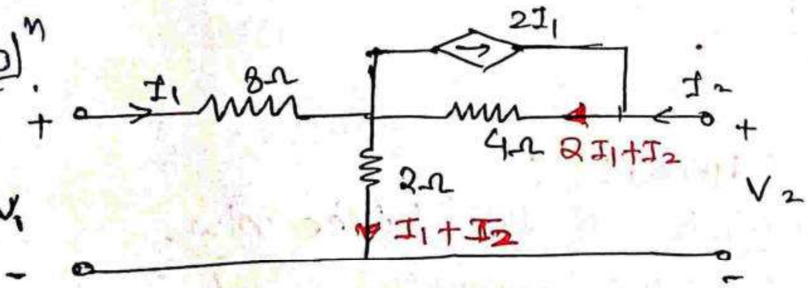
Ex.



Q. find Z-Parameters.



Sol^n



$$V_1 = 8I_1 + 2(I_1 + I_2)$$

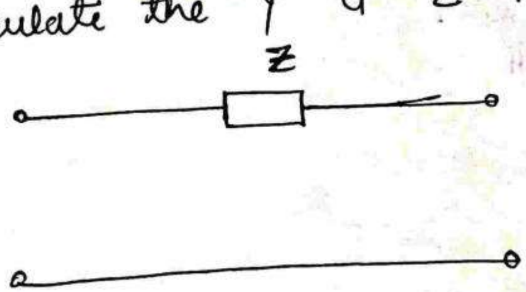
$$V_1 = 10I_1 + 2I_2 \quad \text{— (1)}$$

$$V_2 = 4(2I_1 + I_2) + 2(I_1 + I_2)$$

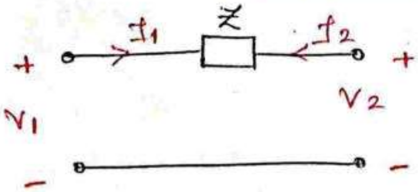
$$V_2 = 10I_1 + 6I_2 \quad \text{— (2)}$$

$$[Z] = \begin{bmatrix} 10 & 2 \\ 10 & 6 \end{bmatrix}$$

Q. Calculate the Y & Z Parameters of below given n/w



Sol^n.



- $I_1 = \dfrac{V_1 - V_2}{Z}$

$I_1 = \dfrac{1}{Z}V_1 - \dfrac{1}{Z}V_2$ —①

$[Y] = \begin{bmatrix} \frac{1}{Z} & \frac{-1}{Z} \\ \frac{-1}{Z} & \frac{1}{Z} \end{bmatrix}$

- $I_2 = -I_1$

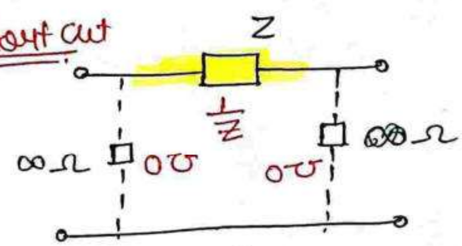
$I_2 = -\dfrac{1}{Z}V_1 + \dfrac{1}{Z}V_2$ —②

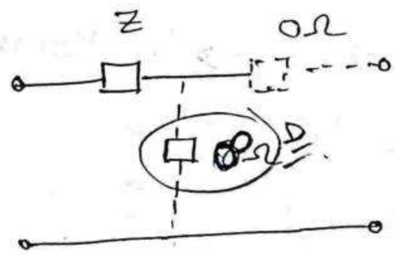
- $[Z] = [Y]^{-1}$

$|Y| = \dfrac{1}{Z^2} - \dfrac{1}{Z^2} = 0$

$\therefore$ $[Y]^{-1}$ & hence $[Z]$ parameter will not exist.

Short cut





$Y_{11} = \dfrac{1}{Z}$

$Y_{12} = Y_{21} = -\dfrac{1}{Z}$

$Y_{22} = \dfrac{1}{Z}$

$\begin{bmatrix} \frac{1}{Z} & \frac{-1}{Z} \\ \frac{-1}{Z} & \frac{1}{Z} \end{bmatrix}$

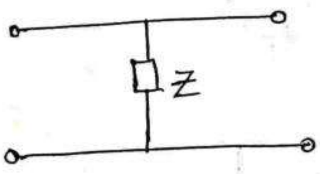
$Z_{11} = \infty$

$Z_{12} = Z_{21} = \infty$

$Z_{22} = \infty$

$\begin{bmatrix} \infty & \infty \\ \infty & \infty \end{bmatrix}$

Q. Calculate the Z & Y Parameters of below given n/w.



Sol^n.

$V_1 = ZI_1 + ZI_2$

$V_2 = ZI_1 + ZI_2$

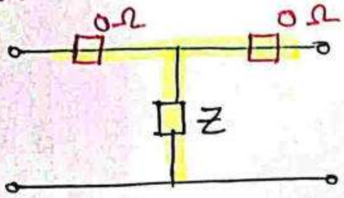
$[Z] = \begin{bmatrix} Z & Z \\ Z & Z \end{bmatrix}$

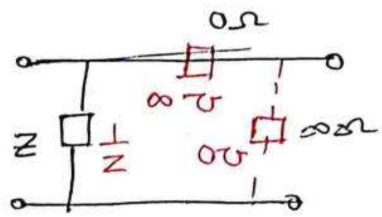
$[Y] = [Z]^{-1}$

$|Z| = Z^2 - Z^3 = 0$

$\therefore$ $[Z^{-1}]$ & hence $[Y]$ Parameters will not exist.

Shortcut





$Z_{11} = Z$ $\quad \begin{bmatrix} Z & Z \\ Z & Z \end{bmatrix}$

$Z_{12} = Z_{21} = Z$

$Z_{22} = Z$

$Y_{11} = \infty$ $\quad \begin{bmatrix} \infty & -\infty \\ -\infty & \infty \end{bmatrix}$

$Y_{12} = Y_{21} = -\infty$

$Y_{22} = \infty$

Interconversion b/w Parameters :

① T-Parameters → Z-Parameter Conversion

$V_1 = AV_2 - BI_2$ — ①

$I_1 = CV_2 - DI_2$ — ②

② → $CV_2 = I_1 + DI_2$

$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2$ — ③

③ – ①

$V_1 = A\left[\frac{1}{C} I_1 + \frac{D}{C} I_2\right] - BI_2$

$= \frac{A}{C} I_1 + \frac{AD}{C} I_2 - BI_2$

$V_1 = \frac{A}{C} I_1 + \frac{AD - BC}{C} I_2$ — ④

$$\boxed{Z_{11} = \frac{A}{C} \quad Z_{12} = \frac{AD - BC}{C} \quad Z_{21} = \frac{1}{C} \quad Z_{22} = \frac{D}{C}}$$

for symmetry

$$Z_{11} = Z_{22}$$

$$\frac{A}{C} = \frac{D}{C}$$

$$\boxed{A = D}$$

for Reciprocity

$$Z_{12} = Z_{21}$$

$$\frac{AD - BC}{C} = \frac{1}{C}$$

$$\boxed{\begin{gathered} AD - BC = 1 \\ |T| = 1 \end{gathered}}$$

② H Parameters $\longrightarrow$ Y Parameters.

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad — ①$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad — ②$$

$$① \rightarrow \quad h_{11} I_1 = V_1 - h_{12} V_2$$

$$I_1 = \frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2 \quad — ③$$

$$③ \rightarrow ②$$

$$I_2 = h_{21} \left[\frac{1}{h_{11}} V_1 - \frac{h_{21}}{h_{11}} V_2 \right] + h_{22} V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{11}} V_2 \quad — ④$$

$$\boxed{\begin{aligned} Y_{11} &= \frac{1}{h_{11}} & Y_{12} &= \frac{-h_{12}}{h_{11}} \\ Y_{21} &= \frac{h_{21}}{h_{11}} & Y_{22} &= \frac{|H|}{h_{11}} \end{aligned}}$$

for symmetry

$$Y_{11} = Y_{22}$$

$$\frac{1}{h_{11}} = \frac{|H|}{h_{11}}$$

$$\boxed{|H| = 1}$$

for reciprocity

$$Y_{12} = Y_{21}$$

$$\frac{-h_{12}}{h_{11}} = \frac{h_{21}}{h_{11}}$$

$$\boxed{h_{12} = -h_{21}}$$

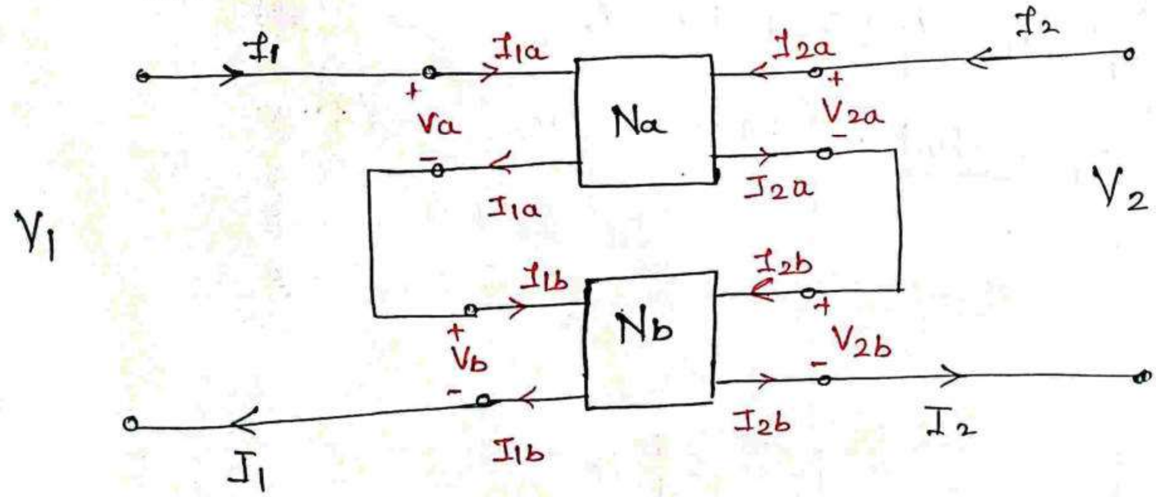
	SYMMETRY	RECIPROCITY
1)	$Z_{11} = Z_{22}$	$Z_{12} = Z_{21}$
	$Y_{11} = Y_{22}$	$Y_{12} = Y_{21}$
2)	$|H| = 1$	$h_{12} = -h_{21}$
	$|G| = 1$	$g_{12} = -g_{21}$
3)	$A = D$	$AD - BC = 1$
		$|T| = 1$
4)	$a = d$	$ad - bc = 1$
		$|t| = 1$

t or abcd

$$\begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = f\begin{pmatrix} V_1 \\ I_1 \end{pmatrix}$$

Interconnection of 2-Port n/ws :

① Series - Series Interconnection —



$$I_1 = I_{1a} = I_{1b}$$

$$I_2 = I_{2a} = I_{2b}$$

$$V_2 = V_{2a} + V_{2b}$$

$$V_1 = V_{1a} + V_{1b}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} + \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \underbrace{\begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix}}_{[Z_a]} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

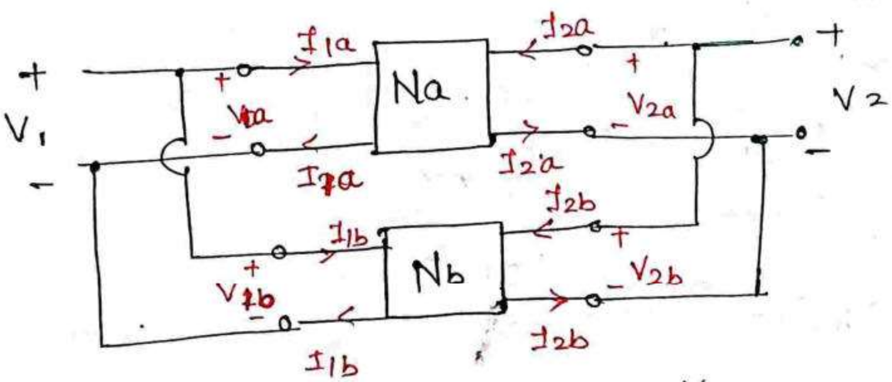
$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = [Z_b] \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Z_a] \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} + [Z_b] \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \{[Z_a] + [Z_b]\} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\boxed{[Z] = [Z_a] + [Z_b]}$$

(2) Parallel - Parallel Interconnection.



$V_2 = V_{2a} = V_{2b}$

$V_1 = V_{1a} = V_{1b}$

$I_2 = I_{2a} + I_{2b}$

$I_1 = I_{1a} + I_{1b}$

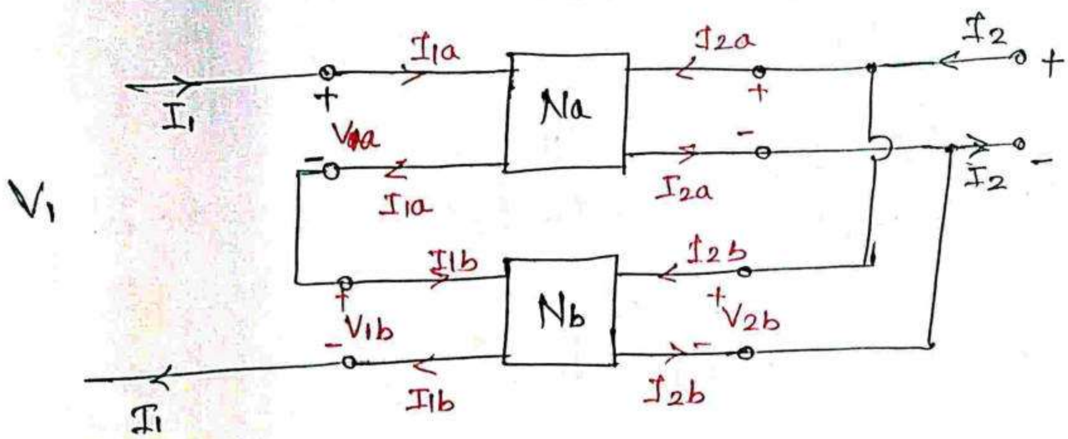
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$$= [Y_a] \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} + [Y_b] \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \{[Y_a] + [Y_b]\} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\boxed{[Y] = [Y_a] + [Y_b]}$$

③ Series- Parallel interconnection.



$I_1 = I_{1a} = I_{1b}$ $\qquad V_2 = V_{2a} = V_{2b}$

$V_1 = V_{1a} + V_{1b}$ $\qquad I_2 = I_{2a} + I_{2b}$

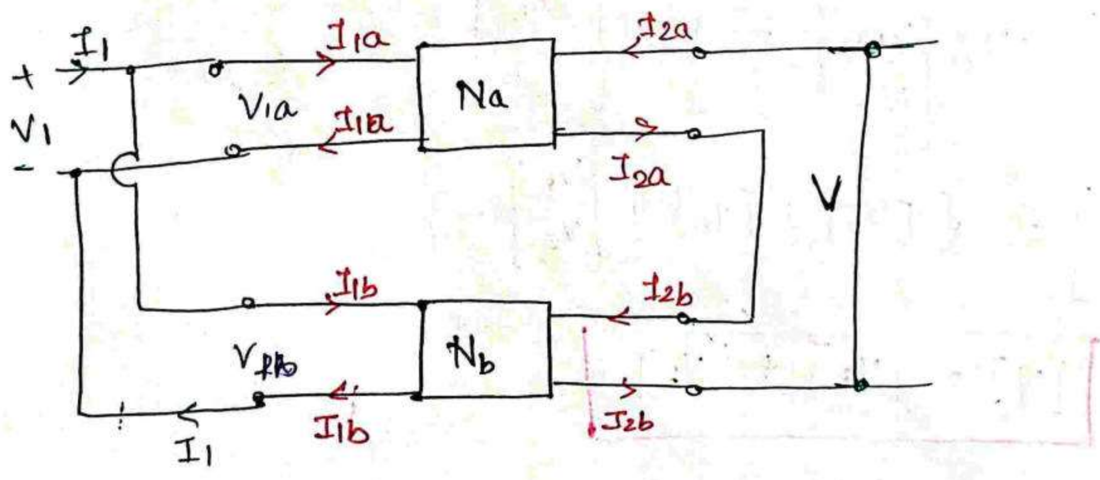
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} V_{1b} \\ I_{2b} \end{bmatrix}$$

$$= [H_a] \begin{bmatrix} I_{1a} \\ V_{2a} \end{bmatrix} + [H_b] \begin{bmatrix} I_{1b} \\ V_{2b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \{ [H_a] + [H_b] \} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\boxed{[H] = [H_a] + [H_b]}$$

④ Parallel- Series Interconnection.



$V_1 = V_{1a} = V_{1b}$ $\qquad$ $I_2 = I_{2a} = I_{2b}$

$I_1 = I_{1a} + I_{1b}$ $\qquad$ $V_2 = V_{2a} + V_{2b}$

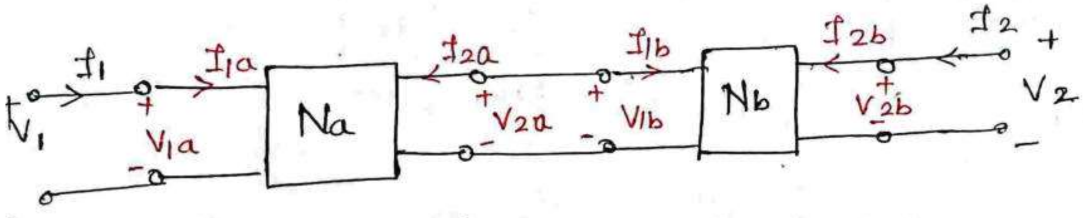
$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{1a} \\ V_{2a} \end{bmatrix} + \begin{bmatrix} I_{1b} \\ V_{2b} \end{bmatrix}$$

$$= [G_a] \begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} + [G_b] \begin{bmatrix} V_{2b} \\ I_{2b} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \{ [G_a] + [G_b] \} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$\boxed{[G] = [G_a] + [G_b]}$$

(5) Cascade Interconnection.



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [T_a] \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [T_a] \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [T_a][T_b] \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [T_a][T_b] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\boxed{[T] = [T_a][T_b]}$$

Bulb

P, V, f

$P = \frac{V^2}{R}$

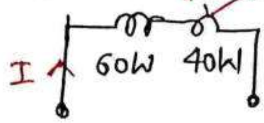
$R = \frac{V^2}{P}$

$\boxed{R \propto \frac{1}{P}}$

- 40W, 220V,
- 60W, 220V

Then, $\underline{R_{40} > R_{60}}$

Q. Which will glow brighter?



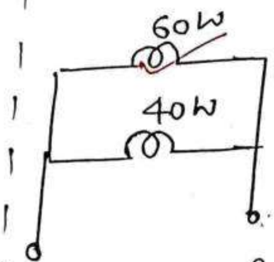
$P_{60} = I^2 R_{60}$

$P_{40} = I^2 R_{40}$

As, $R_{40} > R_{60}$

$\therefore P_{40} > P_{60}$

Q. which will glow brighter.

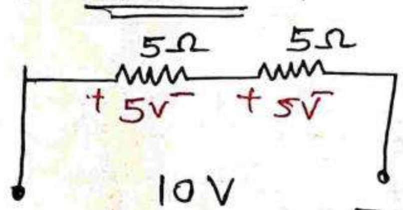


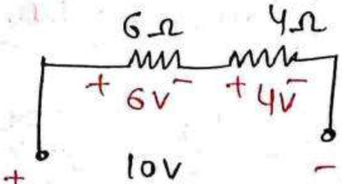
$P_{60} = \frac{V^2}{R_{60}}$

$P_{40} = \frac{V^2}{R_{40}}$

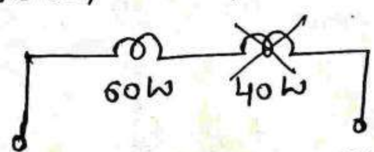
As, $R_{40} > R_{60}$

$\therefore P_{60} > P_{40}$

- 

- 

- 60W, 100V, 40W, 100V



Q→ If we have 2 bulbs rated 60W, 100V & 40W, 100V are connected in series across the power supply of 200V then w/c bulb will get destroy.

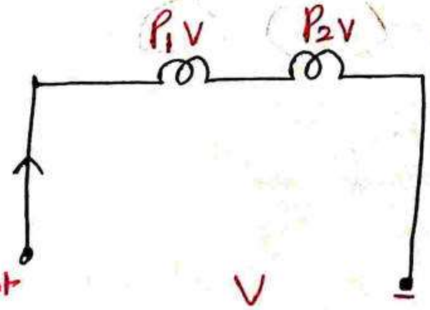
As $R_{40} > R_{60}$

$\therefore V_{40} > V_{60}$

and $\boxed{V_{40} > \frac{V}{2}}$, $\boxed{V_{60} < \frac{V}{2}}$

$\therefore V_{40} > \frac{200}{2}$ & $V_{60} < \frac{200}{2}$

$V_{40} > 100$ & $V_{60} < 100$

- 

Find the total power consumption?

$$P = I^2 R_{eq}.$$

$$P = \frac{V^2}{R_{eq}^2} . R_{eq} \Rightarrow P = \frac{V^2}{R_{eq}} . = \frac{V^2}{R_1 + R_2}$$

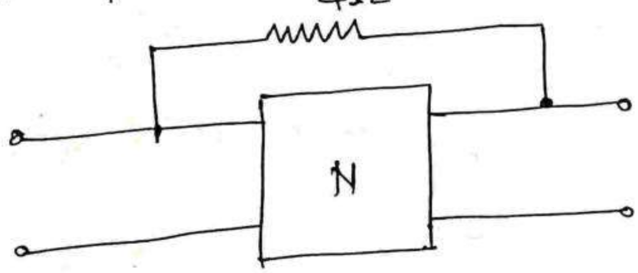
$$P = \frac{1}{\frac{R_1 + R_2}{V^2}} \Rightarrow P = \frac{1}{\frac{R_1}{V^2} + \frac{R_2}{V^2}}$$

$$P = \frac{1}{\frac{1}{\frac{V^2}{R_1}} + \frac{1}{\frac{V^2}{R_2}}} = \frac{1}{\frac{1}{P_1} + \frac{1}{P_2}}$$

$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}$$

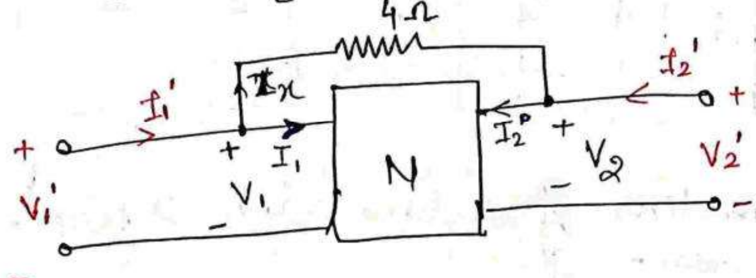
$$\boxed{P = \frac{P_1 P_2}{P_1 + P_2}}$$

Q. Find the Y-Parameters of the below given 2-port n/co



$$[Y_N] = \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}$$

Sol^n



$$I_1 = 2V_1 - V_2 \quad \text{——} \quad (1)$$

$$I_2 = 5V_1 + 3V_2 \quad \text{—} \quad (2)$$

$$I_1' = I_1 + I_x \quad \text{——} \quad (3)$$

$$I_2' = I_2 - I_x \quad \text{——} \quad (4).$$

$$I_x = \frac{V_1 - V_2}{4} \quad \text{--- (5)}$$

(1) & (5) $\longrightarrow$ (3)
$$I_1' = 2V_1 - V_2 + \frac{V_1}{4} - \frac{V_2}{4}$$

$$I_1' = \frac{9}{4} V_1' - \frac{5}{4} V_2' \quad \text{--- (6)}$$

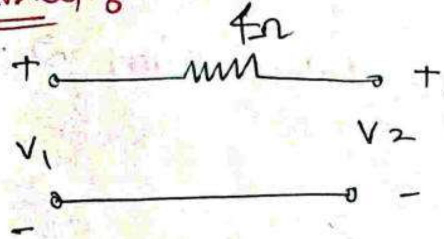
(2) & (5) $\longrightarrow$ (4)

$$I_2' = 5V_1 + 3V_2 - \frac{V_1}{4} + \frac{V_2}{4}$$

$$I_2' = \frac{19}{4} V_1 + \frac{13}{4} V_2 \quad \text{--- (7)}$$

$$[Y] = \begin{bmatrix} \frac{9}{4} & \frac{-5}{4} \\ \frac{19}{4} & \frac{13}{4} \end{bmatrix}$$

II Method :



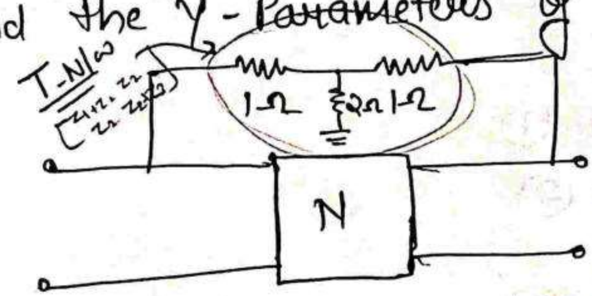
(in Parallel)

$$[Y_1] = \begin{bmatrix} \frac{1}{4} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{1}{4} \end{bmatrix}$$

$$[Y] = [Y_1] + [Y_2]$$
$$= \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix} + \begin{bmatrix} \frac{1}{4} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{9}{4} & \frac{-5}{4} \\ \frac{19}{4} & \frac{13}{4} \end{bmatrix}$$

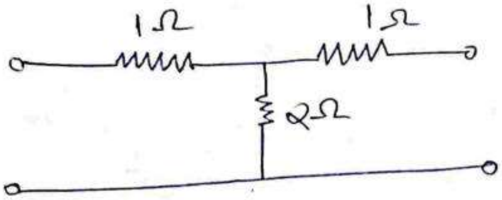
Q. find the Y - Parameters of below given 2 port n/w.

11-1)



$$[Y_N] = \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}$$

Sol^n.

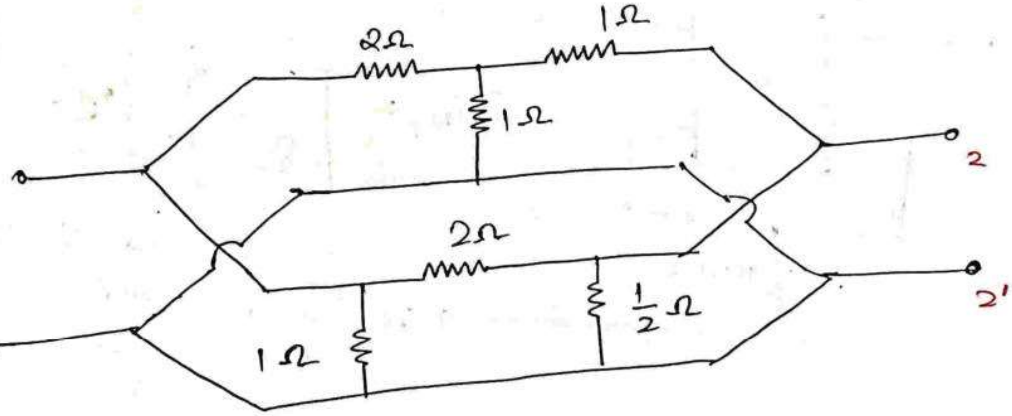


$$[Z_1] = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$

$$[Y_1] = \frac{1}{5}\begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$$

$$[Y] = [Y_1] + [Y_N]$$
$$= \begin{bmatrix} 3/5 & -2/5 \\ -2/5 & 3/5 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} \frac{13}{5} & -\frac{7}{5} \\ \frac{23}{5} & \frac{18}{5} \end{bmatrix}$$

Q.

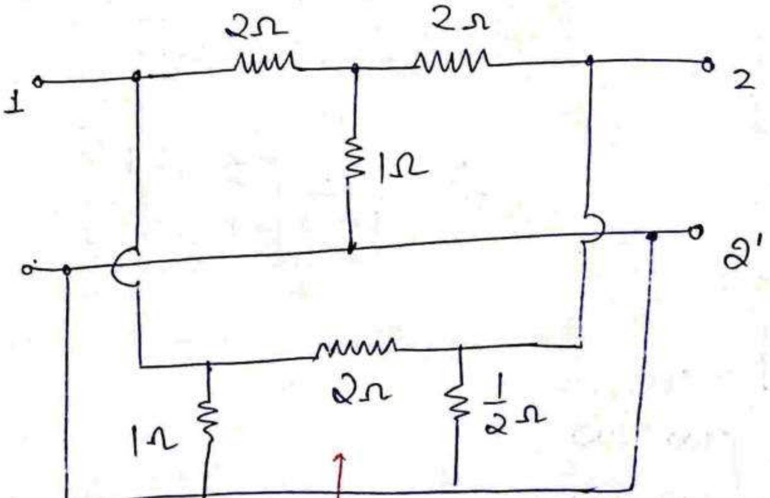


Parallel-Para
⇓
Y
⇓
$Y^{-1} = Z$

Find the Z-Parameters of the below given 2-Port n/w, connection is in

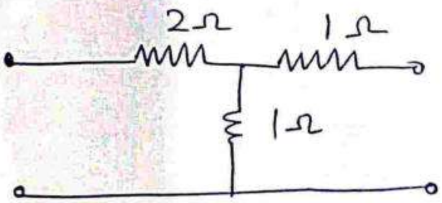
$\|-\|$

So, $[Y] = [Y_a] + [Y_b]$
↓
$[Z] = [Y]^{-1}$



$$[Y_b] = \begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 5/2 \end{bmatrix}$$

Convert in Ʊ
(π–N/w)



$$[Z_a] = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

$$[y_a] = \frac{1}{5}\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

$$[Y] = [y_a] + [y_b]$$

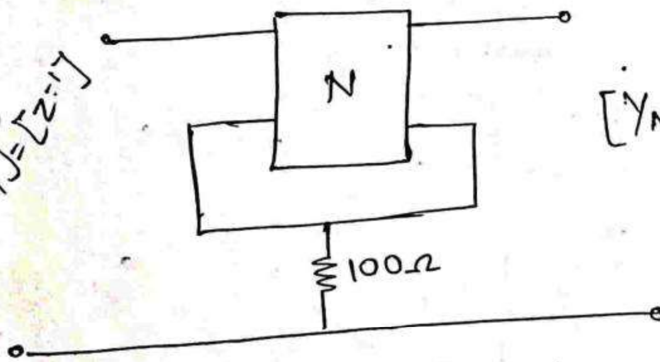
$$[Y] = \begin{bmatrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{bmatrix} + \begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 5/2 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} 19/10 & -7/10 \\ -7/10 & 31/10 \end{bmatrix} \Rightarrow [Z] = [Y]^{-1}$$

$$[Z] = \frac{1}{54}\begin{bmatrix} 31 & 7 \\ 7 & 19 \end{bmatrix}$$

Q. find the y-Parameters of below given 2-port n/w.

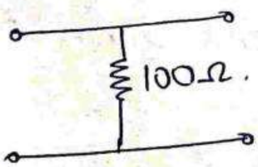
Series-Series
⇓
$[Z]$
$[Y] = [Z^{-1}]$



$$[Y_N] = \begin{bmatrix} 2 & 0 \\ 0 & 10 \end{bmatrix} mS$$

$$[Y_N] = \begin{bmatrix} 2\times10^{-3} & 0\times10^{-3} \\ 0\times10^{-3} & 10\times10^{-3} \end{bmatrix}$$

Sol^n



$$[Z_a] = \begin{bmatrix} 100 & 100 \\ 100 & 100 \end{bmatrix}$$

$$[Z_N] = \begin{bmatrix} 500 & 0 \\ 0 & 100 \end{bmatrix}$$

$$[Z] = [Z_N] + [Z_a]$$

$$= \begin{bmatrix} 500 & 0 \\ 0 & 100 \end{bmatrix} + \begin{bmatrix} 100 & 100 \\ 100 & 100 \end{bmatrix}$$

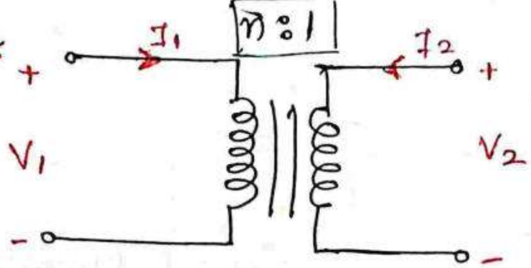
$$= \begin{bmatrix} 600 & 100 \\ 100 & 200 \end{bmatrix}$$

$$[Y] = [Z]^{-1}$$

$$= \frac{1}{600\times200 - 100\times100}\begin{bmatrix} 200 & -100 \\ -100 & 600 \end{bmatrix}$$

$$= \frac{1}{1100}\begin{bmatrix} 2 & -1 \\ -1 & 6 \end{bmatrix}$$

#



For the above n/w, it is not possible to find the impedance & admittance value bcz for an ideal T/F, self and mutual inductance are infinite & hence Z & Y parameters are not defined.

- $$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$\frac{V_1}{V_2} = \frac{n}{1}$$

$$V_1 = nV_2 \quad \text{— (1)}$$

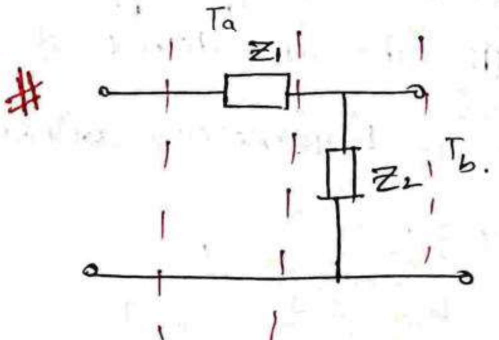
- $$\frac{I_1}{I_2} = \frac{-N_2}{N_1}$$

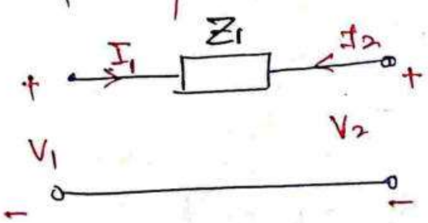
$$\frac{I_1}{I_2} = \frac{-1}{n}$$

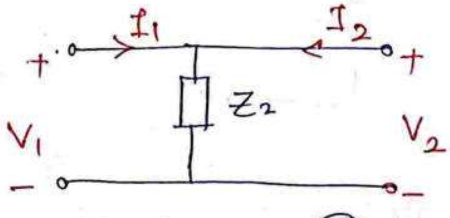
$$I_1 = -\frac{1}{n} I_2 \quad \text{— (2)}$$

$$\boxed{[T] = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}}$$ when $n:1$

#







$$V_1 = I_1 Z_1 + V_2$$

$$V_1 = V_2 - Z_1 I_2 \quad \text{— (1)}$$

$$I_1 = -I_2 \quad \text{—— (2)}$$

$$\boxed{[T_a] = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}}$$

$$V_1 = V_2 \quad \text{— (3)}$$

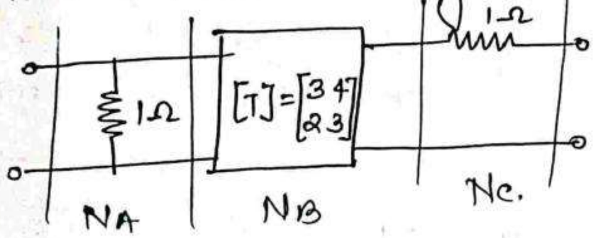
$$I_1 = \frac{V_2}{Z_2} - I_2 \quad \text{— (4)}$$

$$\boxed{[T_B] = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{bmatrix}}$$

$$[T] = [T_a] \cdot [T_b]$$

$$= \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 0 \end{bmatrix} = \begin{bmatrix} 1+\frac{Z_1}{Z_2} & Z_1 \\ \frac{1}{Z_2} & 1 \end{bmatrix}$$

Q. find the T-Parameters of the below given T-Parameters:



$$[T] = [T_a] \cdot [T_b] \cdot [T_c]$$

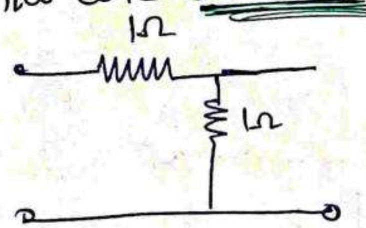
$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 5 & 12 \end{bmatrix}$$

Q. The T-Parameters of a n/w consisting of only impedances are $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$. If ~~all~~ the impedances of n/w are doubled then the new T-Parameters will be:

Sol^n

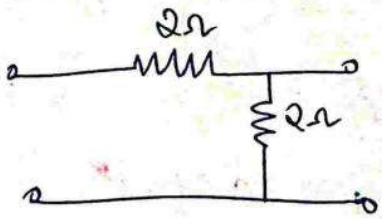
Let.



$$[T] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$A = 2$
$B = 1$
$C = 1$
$D = 1$

Now double all.

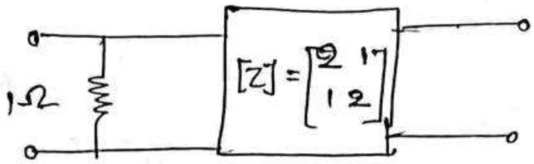


$$[T] = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ \frac{1}{2} & 1 \end{bmatrix}$$

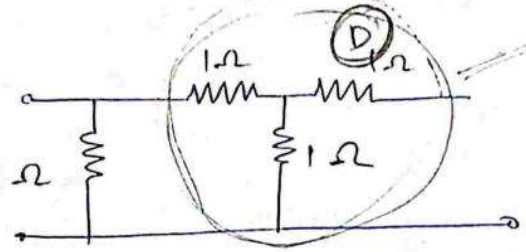
$A = 2$
$B = 2$
$C = \frac{1}{2}$
$D = 1$

A & D → same
B → Double
C → Half.

Q. Find the transmission parameters of below given 2-port N/w.



Z-Parameters are given so we can convert them in T-N/w.

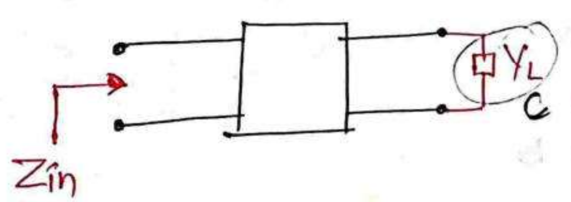


$$[T] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$$

GYRATOR —

- It is a 4 terminal or 2 port device.



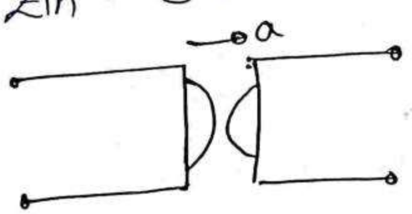
$$\boxed{Z_{in} = a^2 Y_L}$$

Gyration resistance. $a = 1\Omega$

- $Z_s = \frac{1}{CS}$

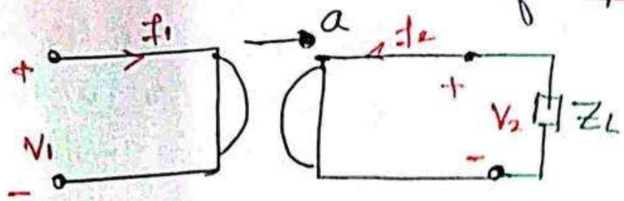
$Z_{in} = a^2 Y_L = \frac{a^2}{Z_L}$

$Z_{in} = S a^2 C$



- Direction of Gyration is important & it tells us in which direction we have to transmit the signal.
- It acts as an Impedance inverter.
- If load is capacitive then i/p impedance is inductive in nature & vice-versa.

• Gyrator is made up of op-amp & RL elements



(2) & (3) → (1)

$$Z_{in} = a^2 Y_L$$

$$Z_{in} = \frac{a^2}{Z_L} \quad \text{— (1)}$$

$$Z_{in} = \frac{V_1}{I_1} \quad \text{— (2)}$$

$$V_2 = -I_2 Z_L$$

$$Z_L = \frac{V_2}{-I_2} \quad \text{— (3)}$$

$$\frac{V_1}{I_1} = \frac{a^2}{\frac{V_2}{-I_2}}$$

$$\frac{V_1}{I_1} = \frac{a^2(-I_2)}{V_2}$$

$$\frac{V_1}{I_1} = \frac{-aI_2}{V_2/a}$$

$$V_1 = -aI_2$$

$$I_1 = \frac{V_2}{a}$$

★ $$\boxed{[T] = \begin{bmatrix} 0 & a \\ \frac{1}{a} & 0 \end{bmatrix}}$$

trick to remember: $V_1 = -aI_2$

$V_2 = +aI_1$

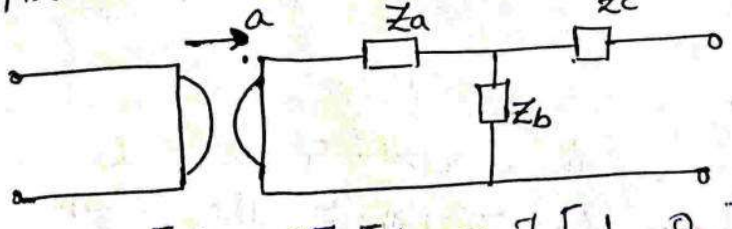
• for reciprocity → $AD - BC$

$= 0 - a \cdot \frac{1}{a}$

$= -1$

∴ Gyrator is non-reciprocal device.

Q. Find T-Parameters.



$$[T] = \begin{bmatrix} 0 & a \\ \frac{1}{a} & 0 \end{bmatrix} \begin{bmatrix} 1 & Z_a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_b} & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_c \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & a \\ \frac{1}{a} & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_c \\ \frac{1}{Z_b} & \frac{Z_c}{Z_b}+1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & a \\ \frac{1}{a} & 0 \end{bmatrix} \begin{bmatrix} \frac{Z_a+Z_b}{Z_b} & \frac{Z_aZ_b+Z_bZ_c+Z_cZ_a}{Z_b} \\ \frac{1}{Z_b} & \frac{Z_b+Z_c}{Z_b} \end{bmatrix}$$

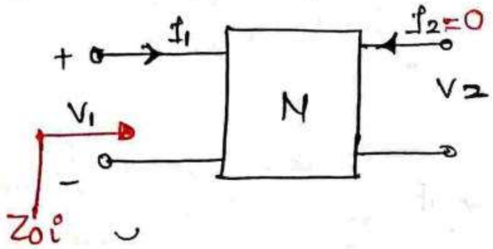
$$[T] = \begin{bmatrix} \frac{a}{Z_b} & \frac{a(Z_b+Z_c)}{Z_b} \\ \frac{Z_a+Z_b}{aZ_b} & \frac{Z_aZ_b+Z_bZ_c+Z_cZ_a}{aZ_b} \end{bmatrix}$$

Open-ckt & Short-ckt impedances in terms of ABCD parameters :

$$V_1 = AV_2 - BI_2 \quad — \text{①}$$

$$I_1 = CV_2 - DI_2 \quad — \text{②}$$

① Open-ckt input Impedances.

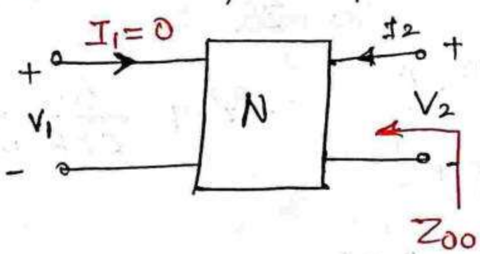


$$Z_{oi} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$= \left.\frac{AV_2-BI_2}{CV_2-DI_2}\right|_{I_2=0}$$

$$\boxed{Z_{oi} = \frac{A}{C}}$$

② ~~Short~~ open-ckt o/p Impedance



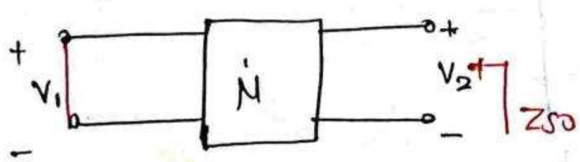
$$Z_{oo} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$\text{②} \rightarrow 0 = CV_2 - DI_2$$

$$Z_{oo} = \frac{V_2}{I_2} = \frac{D}{C}$$

$$\boxed{Z_{oo} = \frac{D}{C}}$$

③ S.C o/p Impedance.



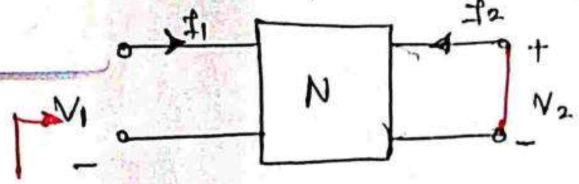
$$Z_{so} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

$$\text{①} \rightarrow 0 = AV_2 - BI_2$$

$$Z_{so} = \frac{V_2}{I_2} = \frac{B}{A}$$

$$\boxed{Z_{so} = \frac{B}{A}}$$

i/p Impedance.

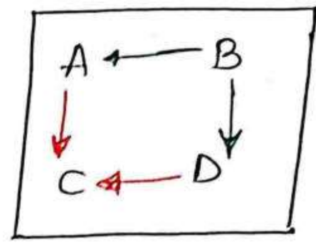


Zsi

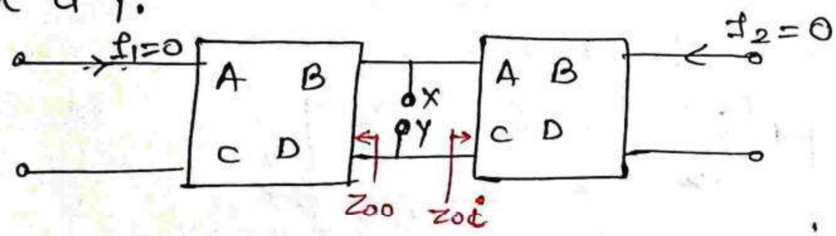
$$Zsi = \frac{V_1}{I_1}\Big|_{V_2=0}$$

$$= \frac{AV_2 - BI_2}{CV_2 - DI_2}\Big|_{V_2=0}$$

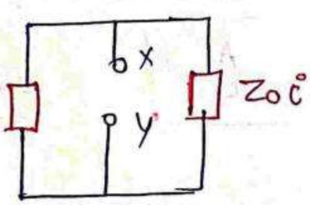
$$\boxed{Zsi = \frac{B}{D}}$$



Q. Find the equivalent impedance b/w the terminals x & y.



Sol^n.



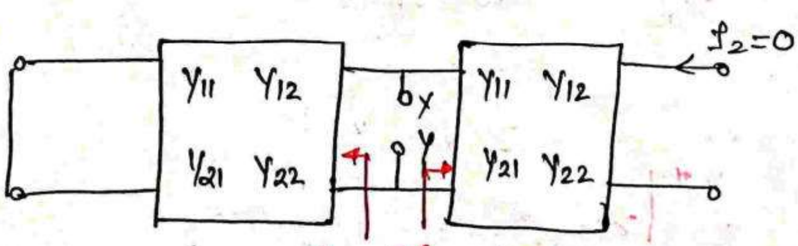
connected in ||

$$Zxy = Zoo \parallel Zoi$$

$$= \frac{Zoo \times Zoi}{Zoo + Zoi}$$

$$Zxy = \frac{\frac{D}{C}\cdot\frac{A}{C}}{\frac{D}{C}+\frac{A}{C}} \Rightarrow Zxy = \frac{AD}{C(A+D)}$$

Q.



$$Zi = \frac{V_1}{I_1}\Big|_{I_2=0}$$

$$= Z_{11}$$

$$= \frac{Y_{22}}{|Y|}$$

$$Zo = \frac{V_2}{I_2}\Big|_{V_1=0}$$

$$= \frac{1}{\frac{I_2}{V_2}\Big|_{V_1=0}}$$

$$= \frac{1}{Y_{22}}$$